DESIGN OF POST-TENSIONED MEMBERS IN BENDING USING ACI 318 –2002 SIMPLIFIED PROCEDURE

1. BACKGROUND

1.1 GENERAL

The following describes the simplified procedure of the American Concrete Institute's (ACI-318-2002) for the design of prestressed concrete sections. The relationships for the simplified procedure is given in Chapter 18 of the code. These are reproduced at the end of this Technical Note for ease of reference. The simplified procedure given in the code is restricted to the cases, where the effective stress in prestressing steel (f_{se}) after allowance for immediate and long-term losses is not less than 50% of its guaranteed ultimate strength ($0.5f_{pu}$)

It uses code specified formulas for the determination of stress in prestressing steel at strength limit state (fse). The rigorous design is based on strain compatibility.

1.2 GEOMETRY

The general geometry of the section considered is shown in **Fig.** 1-1 for a T-section. Inverted L or rectangular sections are treated as special conditions of a T-section in which one, or both of the overhangs are reduced to zero. I-sections at ultimate strength are also treated as T-sections, since the contribution of concrete in tension zone is disregarded.

1.3 MATERIALS AND STRESSES

The stress-strain relationship of the materials used is shown in **Fig.** 1-2 for the general case. At ultimate condition the stresses are idealized using the following relationships:

1.3.1 Concrete

Observe Figure 1-3.

(i) Tension stresses are neglected.

(ii) At ultimate strength compression stresses in concrete may be substituted by a rectangular block of uniform stress extending from the compression fiber over a depth "a".

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$a = \beta_1 * c$ (1-1)

β_1 = Minimum of 0.65 or {0.85 - 0.05*[(f'$_c$ - 4000)/1000]}
but not less than .65 (US Units) (1-2)

β_1 = Minimum of 0.65 or {0.85 - 0.05*[(f'$_c$ - 30)/7]}
but not less than .65 (SI Units) (1-2M)

(iii) The uniform stress over the compression block is assumed as 0.85 f'$_c$

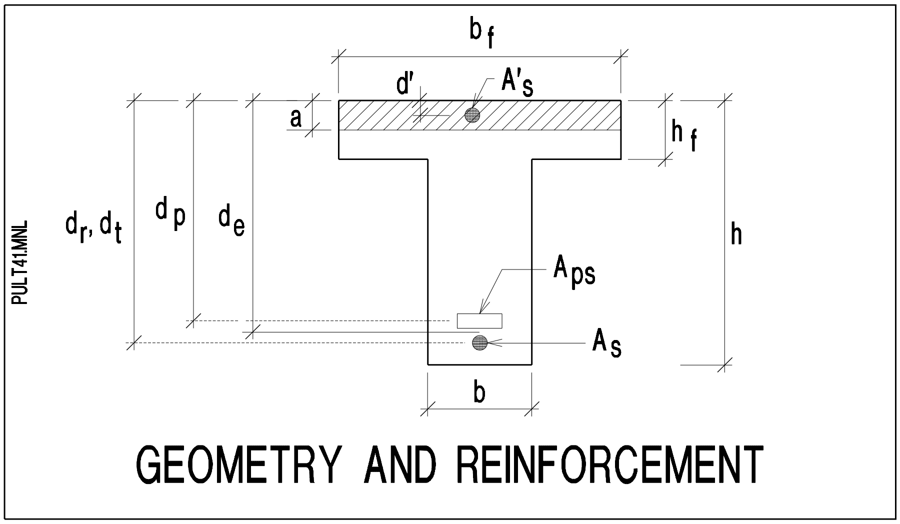

FIGURE 1-1

1.3.2 Non-Prestressed Reinforcement

Non-prestressed reinforcement regardless of its yield stress is referred to as rebar. Rebar may be added to supplement prestressing in developing the required moment resistance.

The stress-strain relationship for rebar is idealized as shown in Figure 1-2(b). If the strain in concrete at the location of the rebar is less than the elastic limit of the rebar material, the rebar will not develop its yield stress. In this case, the calculation uses the stress obtained from the stress-strain diagram of the rebar material.

1.3.3 Prestressing

The stress developed in the prestressing steel at nominal strength is given by f_{ps}. If the effective stress in prestressing (f_{se}) (after allowance for short and long term losses) is not less than 0.5*f_{pu} , the ACI simplified relationships may be used to estimate f_{ps}.

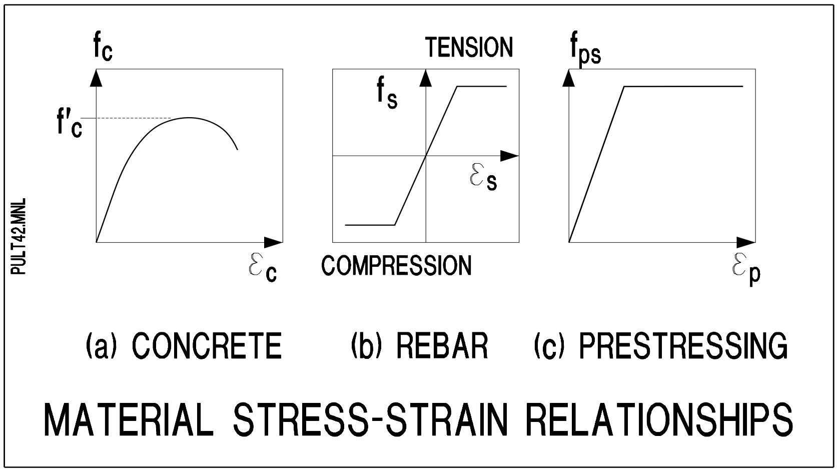

FIGURE 1-2

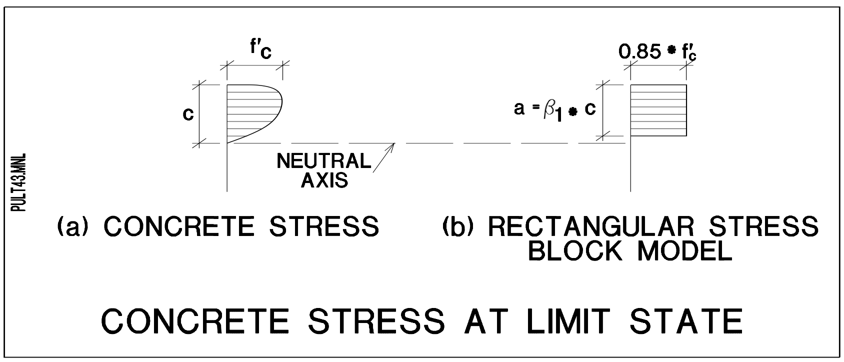

FIGURE 1-3

For grouted tendons, the code uses a parameter γ_p, for the calculation of stress in prestressing steel at strength limit state. γ_p is a constant depending on the material of prestressing tendon.

γ_p = 0.55 for f_{py}/f_{pu} not less than 0.80
γ_p = 0.40 for f_{py}/f_{pu} not less than 0.85
γ_p = 0.28 for f_{py}/f_{pu} not less than 0.90

1.4 REQUIREMENTS

The design requirements are:

(i) The design moment (M_u) must not be less than the moment which the section can develop (M_n = nominal moment) reduced by a strength reduction factor (ϕ). The expression $\phi{*}M_n$ is referred to as design capacity.

$$M_u < \phi{*}M_n \qquad (1\text{-}3)$$

(ii) The section should possess a minimum ductility. In this context ductility is defined as the ratio of rotation of a section at failure (Θ_u at location of plastic hinge) to rotation of the section at its elastic limit (Θ_y, onset of plasticity). Figure 1-4(a) illustrates the definition of ductility as expressed by μ.

Experiments have established that ductility (μ) is primarily a function of the amount and position of prestressing and reinforcement in a section, as well as a section's geometry. The ductility of a section is controlled by the ratio c/d_t and the strength reduction factor ϕ. The minimum ductility required by the code is achieved through the limitation imposed on the ratio c/d_t . For the basic strength reduction factor ($\phi = 0.9$), the ratio of c/d_t is limited to 0.375.

$$c_{max} = 0.375\ d_t \tag{1-4a}$$

$$a_{max} = \beta_1 * c_{max} \tag{1-4b}$$

In a somewhat similar manner, the Canadian code (CSA3-A23.3), the British code (BS 8110), and the European code (EC2) implement the ductility requirement by limiting the maximum depth of the neutral axis c to a fraction of d or h.

Based on the threshold specified for the depth of the neutral axis (c), prior to triggering a reduction in the strength reduction factor (ϕ), six design conditions are identified. These are illustrated in **Fig. 1-5**.

CASE 1: PRESTRESSING ADEQUATE

Case 1 is the condition in which the available prestressing is in excess of that required to resist the design moment M_u, with adequate ductility ($c < 0.375\ d_t$). Clearly, the section is satisfactory as is, no additional rebar is required.

CASE 2: PRESTRESSING PLUS TENSION REBAR

In Case 2, the available prestressing is not adequate to resist the design moment M_u. Rebar A_s is required to supplement the prestressing A_{ps}. The combined areas of A_{ps} and A_s result in ($a < a_{max}$). The larger circle shown in the figure around the rebar represents the maximum area of rebar ($A_{s\ max}$) that would bring the section to its ductility threshold of ($a = a_{max}$).

CASE 3: PRESTRESSING AND TENSION REBAR NOT ADEQUATE

By increasing the applied moment of Case 2, a condition is reached for which the prestressing and the maximum rebar are derived from relationships 1-4 are no longer adequate to develop the required design capacity ϕM_n . In this case, the balance of design capacity must be generated by a force couple resulting from addition of tension and compression rebar. Generally, the area of the added compression steel A'_s will be equal to the added tension steel in excess of $A_{s\ max}$, the exception is when the bars are positioned such that one or both of them would not yield.

CASE 4: OVER-REINFORCED SECTION, DESIGN BASED ON COMPRESSION ZONE ZONE

In Case 4, the amount of available prestressing is excessive, in that ($c > 0.375\ d_t$). As a result the ductility threshold is exceeded . The section can still be considered a satisfactory design, provided that the design moment (M_u) is less than what the design capacity of the compression zone of the section. The design capacity of the compression zone is determined with the depth of the neutral axis assumed at c_{max}. The relationships are (**Figs. 1-6 and 1-7**):

$$\phi M_n = \phi*\{[C_c*(d_p - a/2)] + [C_s*(d_p - d')] + [C_f*(d_p - a/2)]\} \quad (1\text{-}5a)$$

For a T-section where "a" falls in the stem use the following formula:

$$\phi M_n = \phi*\{[C_c*(d_p - a/2)] + [C_s*(d_p - d')] + [C_f*(d_p - h_f/2)]\} \quad (1\text{-}5b)$$

In the above relationships, Cs refers to the component of the force from the compression steel, if available.

CASE 5: PRESTRESSING AND COMPRESSION REBAR

As in Case 4, the design capacity of the compression zone ϕM_n is not adequate to resist the design moment M_u, that is to say ($M_u > \phi*M_n$). In this case, rebar must be added to the compression zone. The design capacity of the section is based on the force developed in the compression zone with a_{max}. The relationships (1-5) apply. The following gives the compression force in its expanded form.

$$C = A'_s*f_{com} + 0.85*(0.5*b*\beta_1*d_r - A_s')*f'_c \quad (1\text{-}6)$$

CASE 6: PRESTRESSING AND COMPRESSION REBAR NOT ADEQUATE

Case 6 is one in which the prestressing alone is in excess of that required to satisfy a_{max} criterion (1-4), and that the maximum compression rebar permissible . An acceptable design can be achieved by resisting the excess moment through addition of rebar for equal tension and compression forces such as to maintain the relationship (1-5).

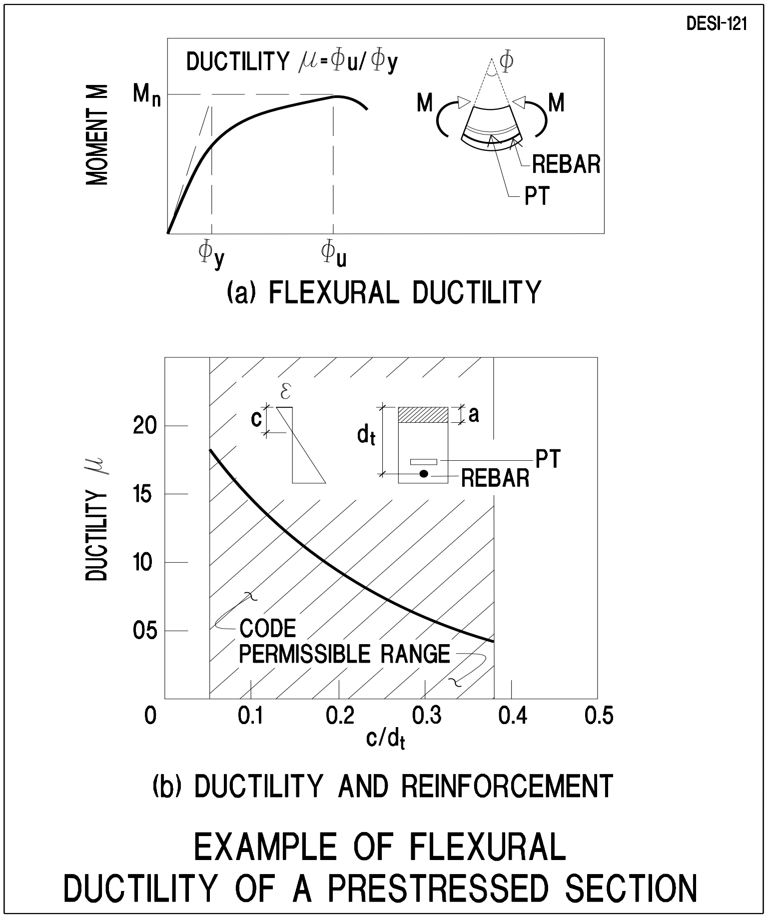

FIGURE 1-4

1.5 RELATIONSHIPS

For given (i) geometry, (ii) material properties, and (iii) amount of prestressing, the design is achieved by obtaining the minimum amount of rebar that develops the required design capacity ϕM_n.

The forces at the strength limit state are defined in **Figs. 1-6** and **1-7**.

(i) Tension equals compression

$$T = C \tag{1-7}$$

Where,

$$T = T_p + T_s \tag{1-8}$$

$$T_p = A_{ps} * f_{ps}$$

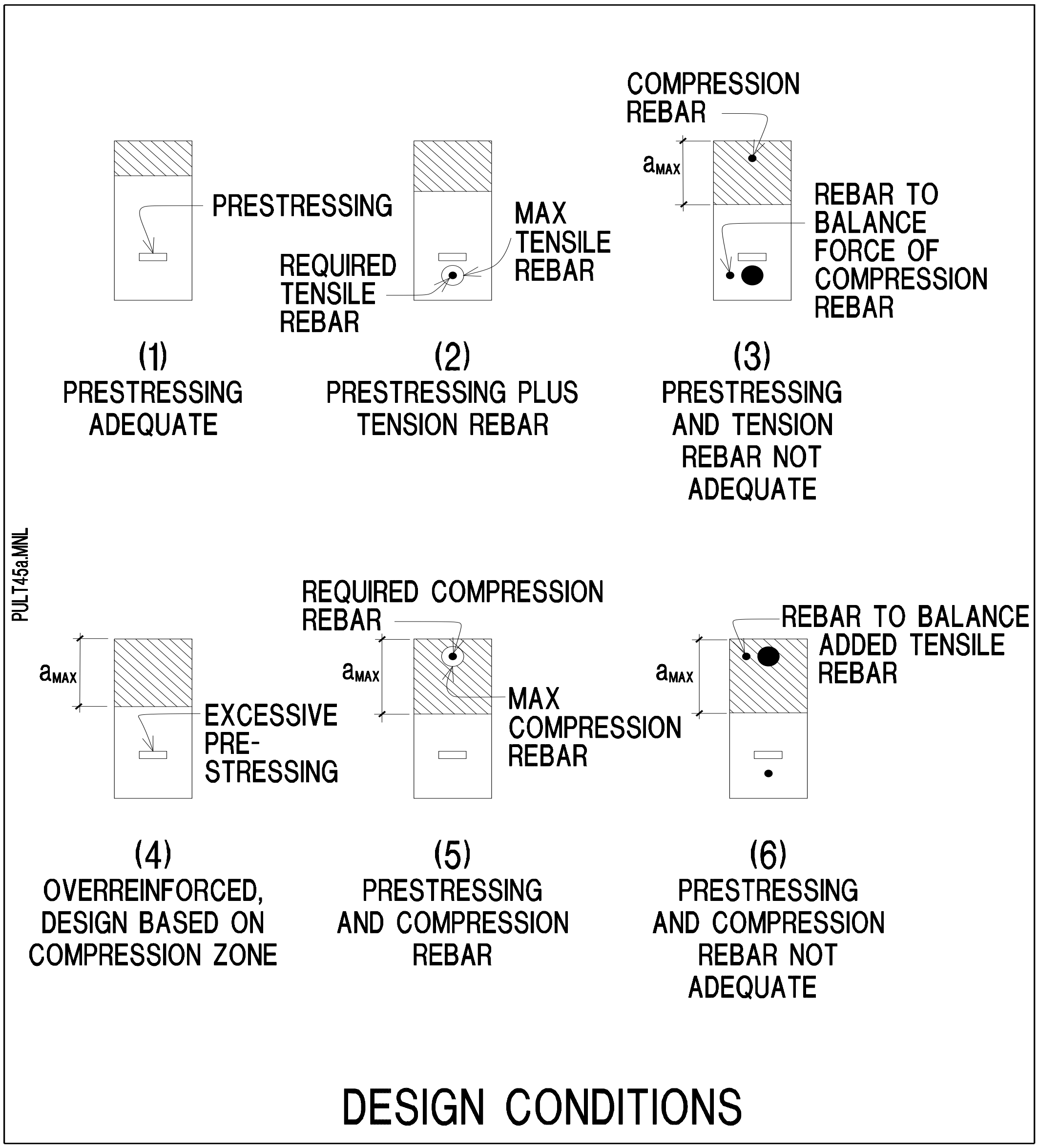

FIGURE 1-5

$$
\begin{aligned}
T_s &= A_s * f_{ten} \\
C &= C_f + C_c + C_s \qquad (1\text{-}9) \\
C_c &= 0.85*(b*a - A'_s)*f'_c \\
C_s &= A'_s * f_{com} \\
C_f &= 0.85*(b_f - b)*a*f'_c
\end{aligned}
$$

(ii) $M_u \leq \phi * M_n$ (1-10)

(iii) $c < 0.375\, d_t$ (1- 11)

(iv) For flanged sections, as shown in **Fig. 1-7**, the contribution of flange overhang to the compression zone is represented by an equivalent compressive force located at the height of the centroid of overhang's compression block. The force developed by the overhang is:

$$C_f = 0.85*(b_f - b)*a*f'_c \tag{1-12}$$

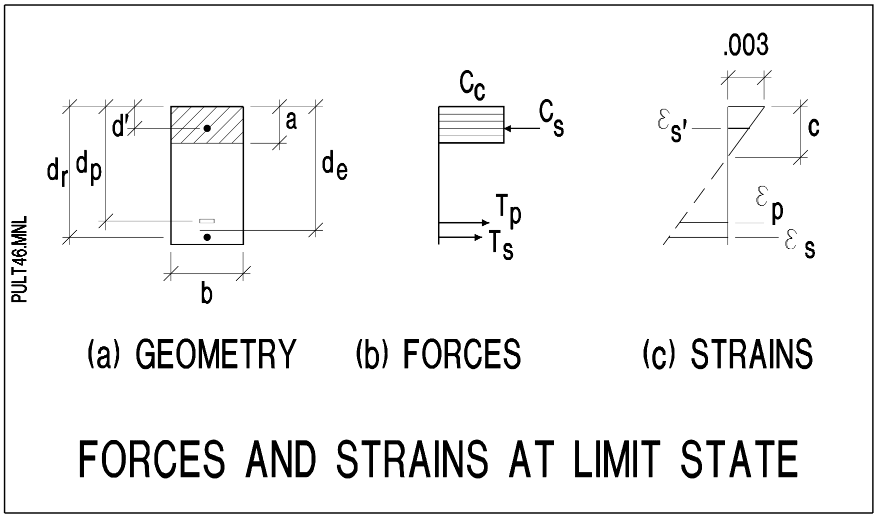

FIGURE 1-6

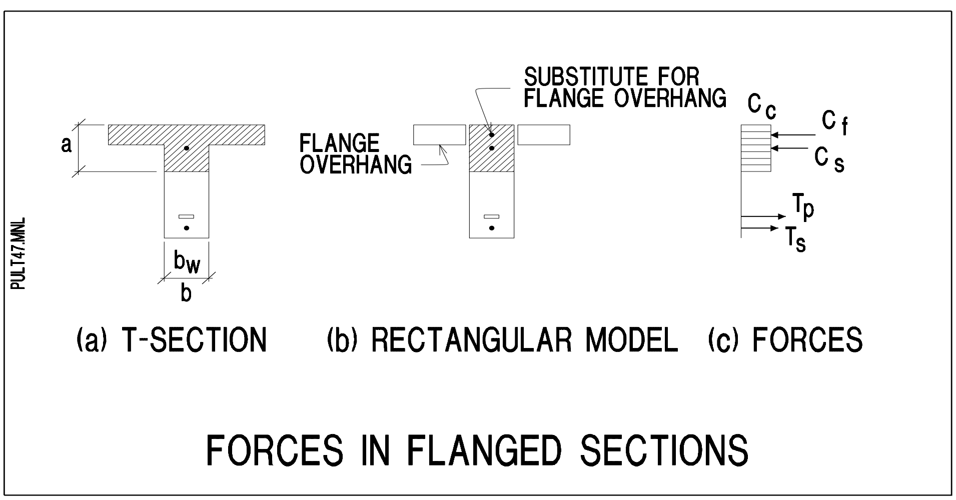

FIGURE 1-7

18.7.2 — As an alternative to a more accurate determination of f_{ps} based on strain compatibility, the following approximate values of f_{ps} shall be permitted to be used if f_{se} is not less than $0.5f_{pu}$.

(a) For members with bonded tendons:

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}(\omega - \omega')\right]\right\} \qquad (18\text{-}3)$$

If any compression reinforcement is taken into account when calculating f_{ps} by Eq. (18-3), the term

$$\left[\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}(\omega - \omega')\right]$$

shall be taken not less than 0.17 and d' shall be no greater than $0.15d_p$.

(b) For members with unbonded tendons and with a span-to-depth ratio of 35 or less:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{100\rho_p} \qquad (18\text{-}4)$$

but f_{ps} in Eq. (18-4) shall not be taken greater than f_{py}, nor greater than $(f_{se} + 60{,}000)$.

(c) For members with unbonded tendons and with a span-to-depth ratio greater than 35:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{300\rho_p} \qquad (18\text{-}5)$$

but f_{ps} in Eq. (18-5) shall not be taken greater than f_{py}, nor greater than $(f_{se} + 30{,}000)$.

RELATIONSHIPS FROM COMPUTATION OF f_{ps}
FROM ACI 318-02

FIGURE 1-8 ACI-318 RELATIONSHIPS FOR THE AMERICAN UNITS

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shall be taken not less than 0.17 and d' shall be no greater than $0.15d_p$.

| (b) For members with unbonded tendons and with a span-to-depth ratio of 35 or less:

$$f_{ps} = f_{se} + 70 + \frac{f_c'}{100\rho_p} \qquad (18\text{-}4)$$

but f_{ps} in Eq. (18-4) shall not be taken greater than f_{py}, nor greater than $(f_{se} + 420)$.

| (c) For members with unbonded tendons and with a span-to-depth ratio greater than 35:

$$f_{ps} = f_{se} + 70 + \frac{f_c'}{300\rho_p} \qquad (18\text{-}5)$$

but f_{ps} in Eq. (18-5) shall not be taken greater than f_{py}, nor greater than $(f_{se} + 200)$.

RELATIONSHIPS FROM COMPUTATION OF f_{ps}
FROM ACI 318M-02

FIGURE 1-9 ACI-318 RELATIONSHIPS FOR SI UNITS

1.7 NOTATION

a	=	depth of compression block;
A_{ps}	=	area of prestressing;
A_s	=	area of tension steel;
A'_s	=	area of compression steel;
b	=	width of rectangular beam, or stem of T-section;
b_f	=	flange width of T-section;
c	=	depth of neutral axis from compression fiber;
C	=	total compression force;
C_c	=	compression force due to concrete;
C_f	=	compression force in flange overhang of T- or inverted L-beam;
C_s	=	compression force due to compression rebar;
d'	=	distance from extreme compression fiber to centroid of compression reinforcement;
d_c	=	distance from extreme compression fiber to centroid of total compression block;
d_e	=	distance from extreme compression fiber to centroid of the resultant tension force;
d_p	=	distance from extreme compression fiber to centroid of prestressed reinforcement;
d_r	=	distance from compression fiber to centroid of tensile rebar;
f'_c	=	28-day compressive strength of concrete;
f_{com}	=	stress in compression rebar at its centroid;
f_{ps}	=	stress in prestress reinforcement at nominal strength;
f_{pu}	=	specified tensile strength of prestressing tendons;
f_{py}	=	specified yield strength of prestressing tendons;
f_{se}	=	effective stress in prestress reinforcement (after allowance for all prestress losses);
f_{ten}	=	stress in tension rebar at its centroid;
f_y	=	specified yield reinforcement of nonprestressed reinforcement;
h	=	overall thickness of member;
h_f	=	thickness of flange of T- or inverted L-beam;
M_u	=	factored moment;
M_n	=	nominal moment;

T	=	total tension force;
T_p	=	tension force due to prestressing;
T_s	=	tension force due to tensile rebar;
β_1	=	factor (= a/c);
γ	=	factor for type of prestressing material;
ε_c	=	concrete's strain;
ε_p	=	strain in tendon at its centroid;
ε_s'	=	strain in compressive steel at its centroid;
ϕ	=	strength reduction factor;
μ	=	ductility factor;
ρ	=	ratio of nonprestressed tension reinforcement (=$A_s/(b*d_r)$);
ρ'	=	ratio of nonprestressed compression reinforcement (=$A_s'/(b*d_r)$);
ρ_p	=	ACI definition (=$A_{ps}/(b*d_p)$);
ω	=	$\rho*f_y/f'_c$
ω'	=	$\rho'*f_y/f'_c$
ω_p	=	ρ_p*f_{ps}/f'_c

1.8 REFERENCE

1. Building Code Requirements for Reinforced Concrete (ACI-318-2002), American Concrete Institute, Detroit, Michigan, 2002